

The methods to detect vacuum polarization by evanescent modes

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We propose the evanescent-mode-sensing methods to probe the electrodynamics (QED) vacuum polarization. From our methods, high-sensitivity can be achieved even though the external field is much smaller than the Schwinger critical field and may be realizable in contemporary experimental conditions. The methods are based on the effect of phase change and time delay of evanescent wave which is transmitted in QED vacuum. These methods can also be widely used in sensitive probing of tiny dissipation in other fields.

Vacuum is one of the most fundamental concepts in all quantum fields of high-energy physics[1], condensed-matter physics[2], statistical optics[3], etc., since all excitations are from the vacuum and determined by vacuum properties in some way. Modern vacuum concept is started from quantum electrodynamics (QED), which describes the interaction between light and matter (including vacuum), and has been widely and continually studied both experimentally and theoretically[4–9]. According to QED, the vacuum becomes weakly anisotropic, dispersive, dissipative and even nonlinear optical medium, when there is an external electric field and its strength is approaching the Schwinger critical value $E_c \simeq 10^{18} \text{V/m}$. In other words, the real and imaginary parts of vacuum refractive index could deviate from unit and zero[5, 6], respectively. Physically, the deviation of the imaginary part is mainly from the electron-positron pair generation. However, the electron-positron pair generation, also generally called as *vacuum polarization* (VP) processes[5], which is schematically shown in Fig.1, has not been directly observed for over half century.

VP processes are very important to understand basic quantum processes in many fields, e.g. in condensed matter physics where the “electron-hole” pair generation is widely used in calculating of electron self-energy and electron-phonon interaction[2]. For QED vacuum, the obstacle to observe VP is the very high critical electric field E_c which is beyond the contemporary technical limit. Therefore, it is natural to wonder if we can find an approach to probe VP with external field E_{ext} much smaller than E_c .

On the other hand, evanescent electromagnetic wave is intensively studied recently because of its potential usage in the sensitive detectors and other directions[10–12]. In this work, we propose the evanescent-mode-sensing methods to detect the QED VP, which is based on the measuring the phase change and the time delay of *evanescent waves* in the vacuum. We find that the required external field could be one order weaker than E_c , which may be realizable by contemporary experiments.

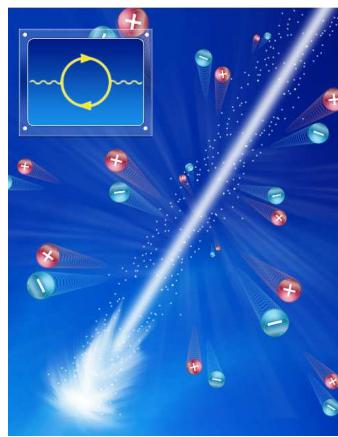


FIG. 1: The schematic picture of vacuum polarization processes with electron-positron pair generation, with which the vacuum becomes dissipative and anisotropic. The insert is the Feynman diagram of the vacuum polarization processes. The fermionic bold line represents the coupling to all orders to the external electromagnetic field.

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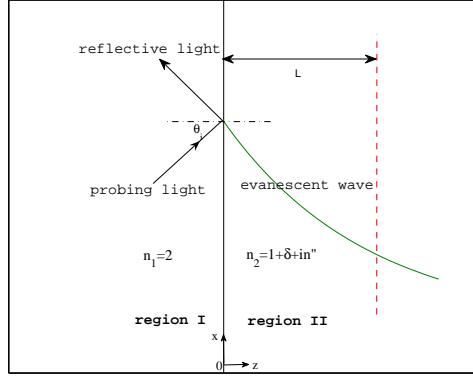


FIG. 2: The schematic diagram of our model.

The idea of this work is from the thought of “dual roles” of real and imaginary parts of refractive index n for the radiative waves or the evanescent waves. Supposing a medium with complex dielectric constant $n = n' + in''$ where $n' \approx 1$ and n'' are extremely small, our goal is to detect the very tiny change of n' and n'' . For radiative waves, n' determines the real part of wavevector $k \approx n'\omega/c$, and we can easily measure the phase change or group delay to detect the change of n' . So, it is natural to choose the radiative wave as the probing light for measuring the index real part change, as what have been done in the experiments to detect the vacuum birefringence effect. On the other hand, for radiative waves, very tiny n'' only causes an extremely small intensity decay, which is very hard to measure in the limited laboratory space. However, for the evanescent waves, the roles of n' and n'' are totally exchanged, i.e., n' dominates the decay rate, while the tiny n'' introduces a real part of wavevector in the decay direction and causes a phase change which is much easier to detect. Further more, we will demonstrate that the tiny imaginary part n'' can also introduce the energy propagation whose energy velocity $v_e \propto n''$ is extremely slow. Such a slow wave can be detected by measuring the delay time τ at a very short distance. The phase change $\Delta\phi$ and the time delay τ can be used for sensitive detecting, especially for the QED VP.

We would like to emphasize the mechanism difference between ours and that in the previous work[12], which is based on the systems with at least “two interfaces” (such as a slab). Such “two-interfaces” structure will generate both evanescent modes $\exp(kx)$ and $\exp(-kx)$ at the same time, and such two evanescent modes can carry electromagnetic energy current[13], which is the essence of “tunneling effect”. So, even the dissipation of material could be neglected (dissipation is truly neglected in that work), the propagation of electromagnetic energy is still available based on that tunneling mechanism. However, in our model, the mechanism is totally different, since there is only one single interface in our model (Fig.2, the details can be seen in the following). The obvious evidence is that, if without dissipation in our “single-interface” model, there will be no the energy current at all in the no-dissipation medium[14], then, the phase change is zero and the energy delay time makes no sense. But, we know there is tiny dissipation in the QED vacuum because of VP, so that the phase change and the energy delay time in our model are not from tunneling effect, but purely from QED vacuum dissipation.

Here we note that, because the probing light is much weaker than the external field in our study, the nonlinear effect is negligible. Since it is a linear problem, all dynamical processes, such as the propagation of envelope fluctuation of the transmitted evanescent wave, can be solved by sum of multi-frequency components. Based on the linear property, we can use Green’s function [14] of multi-frequency components to obtain the strict numerical results, which can be compared with our analytical ones of dynamical process of evanescent wave.

Our model is schematically shown in Fig.2, based on the total internal reflection (TIR) at the interface between a dielectric media n_1 (region I) and vacuum (region II). When the incident angle $\theta_i > \theta_c = \arcsin(1/n_1)$, the TIR will occur and the transmitted wave in the vacuum is the evanescent wave. We choose θ_i is a little larger than θ_c to make sure that almost all frequency components are totally reflected when the incidence is the slowly-varying quasi-monochromatic wave. An interferometer or a photon detector is set at distance L from the interface so that the phase information and intensity change can be detected.

The time-dependent Maxwell equations are given by $\nabla \times \mathbf{E} = -\mu(z)\mu_0 \partial \mathbf{H} / \partial t$ and $\nabla \times \mathbf{H} = \epsilon(z)\epsilon_0 \partial \mathbf{E} / \partial t$, where $\epsilon(z)$ and $\mu(z)$ are the relative permittivity and the relative permeability, respectively, and $c = 1/\sqrt{\epsilon_0\mu_0}$. To obtain the concrete results, the system parameters are chosen as following, the incident angle $\theta_i = 0.1667\pi$, the refractive index of region I $n_1 = \sqrt{\epsilon_1} = 2$, and the vacuum refractive index of region II $n_2 = \sqrt{\epsilon_2\mu_2} = 1 + \delta + in''$, where $\delta \ll 1$ and $n'' \ll 1$ are the real and imaginary index deviations of vacuum, because of VP processes caused by strong external field. If the incident probing light is a plane wave, the transmitted wave in the vacuum region can be generally written in the form $E(x, z, t) = E \exp(ik_z z + ik_{||} r_{||} - i\omega t)$, where $k_{||} = n_1 \sin \theta_i \omega / c$ and $k_z = \sqrt{(n_2 \omega / c)^2 - k_{||}^2}$ are the wave vectors parallel and perpendicular to the interface. For the evanescent

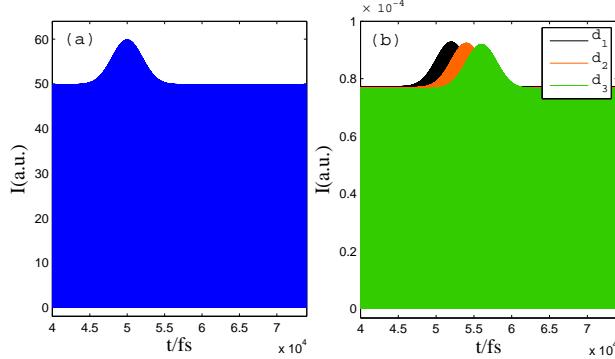


FIG. 3: The irradiance of light I versus time t . (a)The incident probing light at the interface; (b)The transmitted evanescent wave in region II . The black one, the red one and the green one show the evanescent wave at the distance from the interface $d_1 = 0.1\lambda_0$, $d_2 = 0.2\lambda_0$ and $d_3 = 0.3\lambda_0$, respectively, where $\lambda_0 = 600nm$ is the wavelength of the center frequency component of the probing light in the classical vacuum.

wave, k_z is described as:

$$k_z = i \sqrt{(n_1 \sin \theta_i)^2 - (1 + \delta)^2} \frac{\omega}{c} + \frac{n''}{\sqrt{(n_1 \sin \theta_i)^2 - (1 + \delta)^2}} \frac{\omega}{c}. \quad (1)$$

The physical meaning of k_z is very clear that the imaginary part $Im(k_z) = \kappa_z$ corresponds to the exponential decay of the field, and the real part $Re(k_z)$ causes a phase change because of VP. The phase change at distance $z = L$ is

$$\Delta\phi = Re(k_z)L \propto n''L \quad (2)$$

which could be measured by interferometers[15].

Besides the phase change $\Delta\phi$, with the same model as shown in Fig.2, there is another way to detect the tiny n'' by measuring the time delay of irradiance[16] fluctuation of the evanescent wave. The physical process can be explained in the following way. First, we suppose that the incident probing light is not a plane wave anymore, but with a slow intensity fluctuation which is proportional to the irradiance fluctuation, as shown in Fig.3(a), then the question is “What will happen for the evanescent wave in region II ?” From the strict Green’s function method with physical dissipation and dispersion, it is found that the fluctuation will propagate on the evanescent wave from the interface to far away, as shown in Fig.3(b). So, we can measure the time delay τ of the fluctuation propagation on the evanescent wave to detect the VP effect. The propagation speed of irradiance fluctuation can be obtained by the energy velocity v_e which is defined as: $v_e = |\vec{S}_z|/W$, where $\vec{S}_z = \frac{1}{2}Re(E \times H^*)|_z$ is the averaged Poynting vector along z direction, and $W \simeq \frac{1}{4}(\epsilon_0|E|^2 + \mu_0|B|^2)$ is the local energy density of the electromagnetic wave. In our model, one can obtain the energy velocity as:

$$v_e = \chi \cdot n'' \quad (3)$$

with $\chi \simeq c / [(n_1 \sin \theta_i)^2 \sqrt{(n_1 \sin \theta_i)^2 - (1 + \delta)^2}]$, when the dissipation and dispersion are very weak[17]. The physical meaning of v_e can be understood as the “propagation” speed of electromagnetic wave irradiance fluctuation of the evanescent wave, which can be measured by the irradiance measurement technology[16]. Here we note that with tiny n'' the expected energy velocity is much smaller than c , and hence causality is not violated.

Hence, experimentally the time delay τ of the irradiance fluctuation at distance L can be measured:

$$\tau = L/v_e \propto 1/n''. \quad (4)$$

Since it is *near field* phenomenon, the detecting should be very near the interface. For the VP effect, since n'' is extremely small, the “propagation” speed of the irradiance fluctuation is so slow that τ is long enough for detecting even in a very short distance L , i.g. τ gets to pico-second level when the distance is one tenth of the wavelength $L = 60nm$.

Therefore, either the phase change $\Delta\phi$ or the time delay τ are very sensitive for n'' , and the evanescent wave is a good candidate to probe the VP effect. Here, we note that the famous Kramers-Kronig relations, which shows the confinement of causality limit, still fit for QED vacuum[5]. Hence, the direct observation of imaginary part of vacuum index also confirms the dispersive property of QED vacuum.

Next, we will quantitatively study the detect of QED VP by our methods. Supposing that an external homogeneous constant electric field E_{ext} , which is perpendicular to the xz plane and smaller than the Schwinger critical electric field E_c , is applied to the

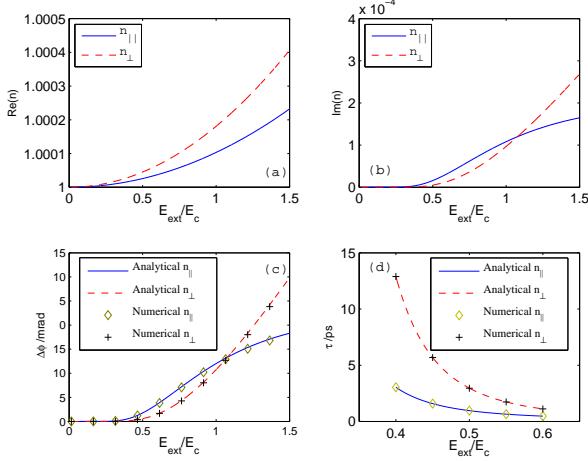


FIG. 4: The refractive index, the phase change and time delay in the QED vacuum with an external electric field E_{ext} . (a)The real part of n_{\parallel} and n_{\perp} versus the external electric field strength; (b)The imaginary part of n_{\parallel} and n_{\perp} versus the external electric field strength; (c)The phase change versus the external electric field strength when the distance from the interface L_p is $6\mu m$; (d)The time delay versus the external electric field strength when the distance from the interface L_τ is $60nm$. Both the results from theory and from Green's function are shown in (c) and (d).

vacuum (region II) only, as shown in Fig.2, then, the optical properties of the vacuum can be described by the Euler-Heisenberg Lagrangian L_{eff} [5, 8]. Physically, the imaginary part of Euler-Heisenberg Lagrangian L_{eff} is related to the imaginary part of VP operator, and therefore corresponds to the *electron-positron pair generation*. This result of QED is well justified not only at zero-temperature but also at finite temperature cases[8]. Consequently, the vacuum refractive index can be deduced from the Lagrangian L_{eff} [5, 8, 9].

In our model, the contribution of the transmitted evanescent wave to L_{eff} is negligible, since its electric field is much weaker than E_{ext} , and furthermore, the external magnetic field is supposed to be zero, thus the vacuum refractive index is determined only by the external homogeneous constant electric field E_{ext} . We use n_{\parallel} and n_{\perp} to refer the effective refractive index of vacuum when the electric field of probing light are parallel and perpendicular to the field E_{ext} , respectively. n_{\parallel} and n_{\perp} can be obtained from the reference [9]:

$$n_{\parallel} = 1 + \frac{2\alpha}{45\pi}y^2 + i \cdot \frac{\alpha}{4\pi} \sum_{n=1}^{\infty} \left(\frac{\pi}{y^2} + \frac{1}{n} \frac{1}{y} \right) \exp(-n\pi/y), \quad (5)$$

and

$$n_{\perp} = 1 + \frac{7\alpha}{90\pi}y^2 + i \cdot \frac{\alpha}{4\pi} \sum_{n=1}^{\infty} \left(\frac{2}{3}\pi + \frac{1}{n} \frac{1}{y} + \frac{1}{n^2} \frac{2}{\pi} \right) \exp(-n\pi/y), \quad (6)$$

where $y = |E_{ext}|/E_c$, and $\alpha \approx 1/137$ is the fine-structure constant. Therefore we have $\delta = Re(n_{\parallel(\perp)}) - 1$, $n'' = Im(n_{\parallel(\perp)})$ for n_{\parallel} and n_{\perp} when we solve the equations such as Eq.(1) in this letter.

The parameters of our model in Fig.2 are chosen as following. The wavelength of probing light is $\lambda_0 = 600nm$, the dielectric constant in the region I is $\varepsilon = 4$, and the incident angle is $\theta_{inc} = 0.1667\pi$, which is a little larger than critical angle $\theta_c = \pi/6$ of TIR, so that the field in vacuum is evanescent. The distance L for the phase detecting is $L_p = 6\mu m = 10 \times \lambda_0$, while for the time delay detecting is $L_\tau = 60nm = 0.1 \times \lambda_0$, respectively. The QED theoretical results of real and imaginary part of n_{\parallel} and n_{\perp} are shown in Fig.4(a) and Fig.4(b), respectively. Bring these results into Eq.(2) and Eq.(4), the phase change $\Delta\phi$ and the delayed time τ can be obtained, which are shown in Fig4(c) and Fig.4(d), respectively. Numerically, the phase change with plane wave incidence and the time delay of local amplitude maximum are calculated by Green's function method, which are also shown in Fig.4(c) and Fig.4 (d). Comparing the analytical results from Eq.(2) and Eq.(3) and numerical results, we can find that they agree with each other very well.

Next, we will analyze the possibility to observe the VP effect in experimental conditions. The recent experimental advances[18] have raised hopes that lasers may achieve fields just one or two orders of magnitude below the Schwinger critical field strength. In this case $E_{ext} \sim 0.1E_c$, from our numerical and analytical results in Fig.4, we can see the $\Delta\phi$ can get to

$\sim 10^{-1} \text{mrad}$ order, which are in measuring limit of contemporary interferometer [15]. Very recently, it is supposed that the electric field E could be effectively amplified 4 times larger by coherent constructive interference of laser beams[7]. If E_{ext} can get to $0.5E_c$ by this method, not only $\Delta\phi$ can be one order larger, but also the delay time τ can get to sub pico-second level and may be measured by contemporary photon detectors.

In conclusion, we have investigated the evanescent-mode-sensing methods to directly detect the QED VP based on evanescent wave. Theoretically, we clearly demonstrate that the imaginary part of QED vacuum index, caused by QED VP processes, can generate a phase $\Delta\phi$ and a time delay τ of irradiance fluctuation propagation on the evanescent wave. From Green's function method, we obtain the numerical results of $\Delta\phi$ and τ , which agree with our analytical ones very well. The possibility to directly observe the effects of VP based on evanescent wave is discussed, and it is found that the required external electric field could be much smaller than the Schwinger critical field and maybe realizable in contemporary experiments. Our methods can also be used in other extremely sensitive detections.

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